A multivariate GARCH model with volatility spill-over and time-varying correlations*

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Abstract

There is now a wide array of GARCH models available that are able to capture many important features of a univariate return time series. However, a lot of questions still remain open about which models are suitable for capturing the dynamics of multivariate return time series. This paper introduces a model for asset returns that incorporates joint heteroscedasticity as well as time varying correlations. It nests the model with cross-sectional volatility by Hwang and Satchell (2005) as well as the dynamical conditional correlation framework by Engle (1999).

We fit the model to ten years of data for stocks in the Dow Jones Industrial Average. The empirical results suggest that the average pairwise correlation behaves strongly countercyclical. This means that the benefits of diversification go down exactly when they are most desirable, which might serve as an explanation why the volatility smile in index options tends to be more pronounced than in individual stocks options.

JEL Classification Codes: C32, G0, G1
Keywords: Dynamic Correlation, Multivariate GARCH, Volatility

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1 Introduction

Many financial time series such as stock returns and exchange rates, exhibit changes in volatility over time. These changes tend to be serially correlated and in the generalized autoregressive conditional heteroscedasticity, or GARCH model, developed by Engle (1982), and Bollerslev (1986), such effects are captured by letting the conditional variance be a function of the squares of the previous returns and past variances. A wide range of GARCH models have now appeared in the econometric literature; see Bollerslev, Chou, and Kroner (1992) for a survey.

There is now a wide array of GARCH models available that are able to capture many important features of a univariate return time series. On the other hand, a lot of questions remain about which models are suitable for capturing the dynamics of multivariate return time series. Hwang and Satchell (2005) examine cross-sectional volatility, which is a special case of common heteroscedasticity in asset returns. On the other hand, Engle (1999), and Engle and Sheppard (2001) introduce a multivariate GARCH model with time-varying correlations. However, few researchers have examined parsimonious models that are able to capture both of these features of multivariate asset returns for a large number of assets. The major problem is the number of parameters to be estimated, for example the original *vech* model for m assets has $O(m^4)$ and a standard BEKK model $O(m^2)$ parameters.

This paper introduces a parsimonious model for asset returns that incorporates joint heteroscedasticity as well as time varying correlations. It nests the models by Hwang and Satchell (2005) as well as Engle (1999) and allows to examine the joint correlation and volatility dynamics. For example, a priori one would expect correlation to be higher in more volatile times.

The paper is organized as follows. Section 2 introduces the GARCH(p,q)-VS(k)-DCC(r,s) model (for GARCH model of order (p, q) with volatility spill-over of order k and dynamic conditional correlation of order (r, s)). Section 3 examines some empirical properties of the model when fitted to stocks in the Dow Jones Industrial Average. Section 4 takes a look at the differences between conditional and contemporaneous correlation. Section 5 concludes and outlines some areas for future research.

2 Model Setup and Estimation

The proposed multivariate GARCH model assumes that the returns of m available assets are conditionally multivariate normal with mean zero[1] and covariance matrix H_t. Formally,

$$r_t|\mathcal{F}_{t-1} \sim N(0, H_t)$$

[1]Since we are not interested in the behavior of the time-varying mean returns and because we are using daily data, we simply set the expected return equal to zero. However, our setup could be easily extended to incorporate a separate model for the conditional mean.

and

$$H_t = D_t R_t D_t,$$

where D_t is an $m \times m$ diagonal matrix of time varying standard deviations of individual assets returns with $\sqrt{h_{i,t}}$ on the $i-th$ diagonal, and R_t is the time varying correlation matrix.

2.1 Volatility Spill-over

Jones (2001), and Connor and Linton (2001) find that common volatility is an important source of individual stock volatility. They show that on average 12%-16% of individual volatility is explained with market common volatility. We therefore propose a volatility model of the form

$$h_{i,t} = \kappa_i r_{i,t-1}^2 + \tilde{\lambda}_i h_{i,t-1} + \sum_{n=1}^{k} \sum_{j \neq i} \tilde{c}_{ij}^n r_{j,t-n}^2, \tag{1}$$

where c_{ij}^n is the variance (or volatility) spill-over from company j to company i at lag n. Will will call this model specification GARCH(1,1)-VS(k)[2]. The very general specification (1) requires $O\left(m^2 k\right)$ parameters to be estimated and quickly becomes infeasible for more than two or three assets. One therefore needs to impose a simple structure on the coefficients c_{ij}^n, for example one could adopt a factor model where volatility spill-over between two companies is proportional to the their common loadings on a small number of factors. This would allow companies in the same sector to exhibit larger co-movements in heteroscedasticity than companies in different sectors.

For the sake of exposition, we adopt a parsimonious specification for volatility spill-over, where $k = 1$ and

$$\tilde{c}_{ij} = \tilde{c}_{ij}^1 = c_i w_j,$$

where w_j is the market capitalization of company j, and where c_i is an asset specific constant measuring the influence of other firm on the volatility of company i. This model specification is motivated by the findings in Conrad, Gultekin, and Kaul (1991), who noted that shocks to large firm returns are important to the future dynamics of their own volatility as well as the volatility of small firm returns. Conversely, shocks to small firms tend to have no impact on the behavior of the volatility of large firms.

It is easy to see that (1) can then be rewritten as

$$h_{i,t} = \kappa_i r_{i,t-1}^2 + \lambda_i h_{i,t-1} + c_i \frac{\left\langle \vec{r}_{t-1}^2, \vec{w} \right\rangle}{\|\vec{w}\|_1}, \tag{2}$$

[2]It is easy to extent this model to include multiple lags of a stock's squared return and volatility, and we would call this a GARCH(p,q)-VS(k) model. However, for the sake of exposition we consider a simple case here, that nevertheless provides a satisfactory fit in most applications.

where $\langle \vec{r}^2_{t-1}, \vec{w} \rangle$ denotes the inner product of the vector $\vec{r}^2_{t-1}$ of individual square stock returns at time $t-1$, and $\vec{w}$ the vector of market capitalizations[3].

In order for conditional volatility in (2) to be always positive, we impose the following conditions on the parameters

$$\kappa_i > 0,\, \lambda_i \geq 0,\, c_i \geq 0 \quad i = 1, \ldots, m. \tag{3}$$

2.2 Time-varying Correlations

Let D_t denote the diagonal matrix of individual stock volatilities σ_i. The proposed dynamic correlation structure is similar to a GARCH(1,1) model:

$$Q_t = (1 - \alpha - \beta)\, \bar{Q} + \alpha \varepsilon_{t-1} \varepsilon'_{t-1} + \beta Q_{t-1} \tag{4}$$

$$R_t = diag\,(Q_t)^{-1} \cdot Q_t \cdot diag\,(Q_t)^{-1} \tag{5}$$

where $\bar{Q}$ is the unconditional covariance of the standardized residuals $\varepsilon_t = D_t^{-1} r_t$. This specification of time-varying correlations was studied extensively by Engle (1999), and Engle and Sheppard (2001), and is commonly referred to as DCC(1,1), for dynamic conditional correlation with lags equal to one[4].

2.3 Estimation

We are going to estimate a GARCH(1,1)-VS(1)-DCC(1,1) model. In matrix notation, the model specification can be written as follows

$$\begin{array}{l} r_t | \mathcal{F}_{t-1} \sim N\,(0, D_t R_t D_t) \\ D_t^2 = diag\,(\kappa_i) \circ r_{t-1} r'_{t-1} + diag\,(\lambda_i) \circ D^2_{t-1} + diag\,(c_i) \circ \langle \vec{r}^2_{t-1}, \vec{w} \rangle \,/\, \|\vec{w}\|_1 \\ \varepsilon_t = D_t^{-1} r_t \\ Q_t = (1 - \alpha - \beta)\, \bar{Q} + \alpha \varepsilon_{t-1} \varepsilon'_{t-1} + \beta Q_{t-1} \\ R_t = diag\,(Q_t)^{-1} \cdot Q_t \cdot diag\,(Q_t)^{-1}\,, \end{array}$$

where $\circ$ is the Hadamard product of two identically sized matrices which is simply computed by element by element multiplication.

The log likelihood for the parameters $\theta = (\kappa_i, \lambda_i, c_i, \omega, \alpha, \beta)$ can be expressed as

$$L\,(\theta |\, \{r_{i,t} : 1 \leq i \leq m, 1 \leq t \leq T\}) =$$

$$= -\frac{1}{2} \sum_{t=1}^{T} \left(m \log (2\pi) + \log |H_t| + r'_t H_t^{-1} r_t \right) =$$

[3]Note that (2) can also be seen as a so called BEKK model (see Engle and Kroner (1995)) with a special structure imposed on the coefficient matrices. However, our model also has a dynamic conditional correlation model layered on top of the volatility model.

[4]Again, this specification is easily extended to include multiple lags of the standardized residuals and of Q_t

$$= -\frac{1}{2}\sum_{t=1}^{T}\left(m\log\left(2\pi\right) + \log\left|D_t R_t D_t\right| + r_t'\left(D_t R_t D_t\right)^{-1} r_t\right) =$$

$$= -\frac{1}{2}\sum_{t=1}^{T}\left(m\log\left(2\pi\right) + 2\log\left|D_t\right| + \log\left|R_t\right| + \varepsilon_t' R_t^{-1}\varepsilon_t\right) =$$

$$= -\frac{1}{2}\sum_{t=1}^{T}\left(m\log\left(2\pi\right) + 2\log\left|D_t\right| + r_t' D_t^{-1} D_t^{-1} r_t - \varepsilon_t'\varepsilon_t + \log\left|R_t\right| + \varepsilon_t' R_t^{-1}\varepsilon_t\right). \quad (6)$$

Let φ denote the parameters governing the volatility dynamics, and ψ the parameters governing the correlation dynamics. The log-likelihood function (6) can be written as the sum of a volatility part and a correlation part:

$$L\left(\theta\right) = L\left(\varphi, \psi\right) = L_V\left(\varphi\right) + L_C\left(\varphi, \psi\right), \quad (7)$$

where the volatility term is

$$L_V\left(\varphi\right) = -\frac{1}{2}\sum_{t=1}^{T}\left(m\log\left(2\pi\right) + 2\log\left|D_t\right| + r_t' D_t^{-1} D_t^{-1} r_t\right),$$

and the correlation term is

$$L_C\left(\varphi, \psi\right) = -\frac{1}{2}\sum_{t=1}^{T}\left(\log\left|R_t\right| + \varepsilon_t' R_t^{-1}\varepsilon_t - \varepsilon_t'\varepsilon_t\right). \quad (8)$$

The volatility part of the likelihood is simply the sum of individual GARCH likelihoods with an additional explanatory variable:

$$L_V\left(\varphi\right) = \sum_{i=1}^{m}\left[-\frac{1}{2}\sum_{t=1}^{T}\left(\log\left(2\pi\right) + \log\left(h_{i,t}\right) + \frac{r_{i,t}^2}{h_{i,t}}\right)\right]$$

which can be maximized by separately maximizing each term. Conditional on the standardized residuals ε_t, the correlation part of the likelihood function can maximized numerically, for example using the Newton-Raphson algorithm. Note that given the value of h_{it}, the squared standardized residuals in (8) do not enter the first order conditions and can therefore ignored.

In summary, the following two-step procedure can be used for obtaining parameter estimates:

1. Fit univariate GARCH models (with an additional explanatory variable) to the return time series of each asset, i.e. solve for

   $$\hat{\varphi} = \arg\max L_V\left(\varphi\right)$$

2. Given the parameter estimate $\hat{\varphi}$ of the volatility dynamics, estimate the correlation dynamics via

   $$\hat{\psi} = \arg\max L_C\left(\hat{\varphi}, \psi\right).$$

Under reasonable regularity conditions, consistency of the first step will ensure consistency of the second step. Engle and Sheppard (2001) give technical conditions that ensure not only consistency but also asymptotic normality of the parameter estimates for a dynamic conditional correlation GARCH model. These conditions do not ensure asymptotic efficiency, however when an estimator is $\sqrt{n}-$consistent, then a fully efficient estimate can be obtained by conducting just one step in the Newton-Raphson algorithm (see Lehman and Casella (1998), theorem 6.4.3).

3 Empirical Results

3.1 Data

The data consists of daily returns of stocks in the Dow Jones Industrial Average Index between January 1, 1996 and December 31, 2005. Note that since the components of the index have changed during the sample period, equities included in the index as of the March 6, 2006 were used. Table 1 shows the composition of the Dow Jones Industrial Average Index as of this date[5].

3.2 Empirical Volatility Dynamics

Table 2 shows the fitted coefficients of the volatility part of the model. First we see that none of the conditions in equation (3) are active. Next, in all but one case the volatility spill-over parameter c_i is statistically significant, which confirms our intuition that stock returns not only exhibit common heteroscedasticity. Our results are in line with Jones (2001), Connor and Linton (2001), and Campell, Lettau, and Xu (2001) who find market volatility is an important component of individual asset volatility.

Figure 1 plots the histogram of standardized residuals $\varepsilon_t = D_t^{-1} r_t$. We see that even though the distribution is no perfectly normal, that the deviation is fairly small. For example, the variance, skewness and kurtosis of the standardized residuals are equal to 1.003, -0.096 and 8.092, respectively.

3.3 Empirical Correlation Dynamics

Table 3 shows the fitted parameters for the correlation part of the model. We see that asset return correlations are highly persistent. Moreover, since the parameter value for α is rather small, the conditional correlation between two companies i and j can only change significantly over a single day if the returns for both assets are large in absolute value. For example, if the stock prices for two companies drop by five standard deviations, then their conditional correlation increases by $0.0004 \cdot 5^2 = 1\%$.

[5] Note that the DJIA is not a value-weighted but a price-weighted index. This means for example that Honeywell International Inc. currently has a larger weight in the index than General Electric Co., even though its market capitalization is about ten times smaller.

Figure 2 shows the average conditional pairwise correlation of the stocks in the Dow Jones Industrial Average, which is defined as

$$\bar{\rho}_t \equiv \frac{2}{m\,(m-1)} \sum_{1\leq i<j\leq m} R_t^{ij}$$

We see that it peaked in the second half of 1997 during the Asian financial crisis, at Russia's default in August 1998, and at the height of the bear market in 2002 and early 2003. Somewhat surprisingly, September 2001 did not seem to have a sustained effect on correlations, as opposed to the wave of defaults and accounting scandals that followed in 2002. On the other had, average correlation bottomed at the height of the bull market in 2000. In summary, asset correlation seems to behave strongly countercyclical, so that the benefits of diversification go down exactly when they are most desirable. As a consequence, the physical distribution of index returns will be negatively skewed, even if individual asset return distributions are symmetric.

The property that correlation in a portfolio goes down when volatility goes up might serve as an explanation for the puzzle that index options tend to exhibit a rather pronounced volatility smile, while individual equity options do not. Bakshi, Kapedia, and Madan (2003) show that skewness of a return distribution under the physical measure $\mathbb{P}$ translates[6] into skewness under the risk-neutral distribution $\mathbb{Q}$, which in turn translates into a volatility smile.

Figure 3 shows the histogram of innovations for the average pairwise correlation $\bar{\rho}_t$. We see that, with a skewness of 5.35 and a kurtosis of 78.6, the distribution of innovations is heavily skewed to the right and fat-tailed. Most of the time the average conditional pairwise correlation hardly moves, but if it moves then the effect will be quite persistent since β, the decay parameter for correlation, is close to 1. It therefore would be interesting instead of (4) and (5) to examine some other specifications of the correlation dynamics, for example

$$Q_t = \omega \bar{Q} + \alpha \varepsilon_{t-1}\varepsilon_{t-1}' + \beta Q_{t-1} + \gamma \varepsilon_{t-1}\varepsilon_{t-1}' 1_{\langle \vec{r}_{t-1}^2, \vec{w}\rangle < 0} \tag{9}$$

$$R_t = diag\,(Q_t)^{-1} \cdot Q_t \cdot diag\,(Q_t)^{-1}\,. \tag{10}$$

Not only might this give a better model for correlations, but also fix some problems with the DCC approach addressed in the next section. We leave this topic open for future research.

4 Contemporaneous vs. Conditional Correlation

As seen in the previous section, somewhat surprisingly, the average conditional correlation of stocks in the DJIA stayed between 0.25 and 0.35 during the whole ten year period

[6] Up to first order in the relative risk aversion parameter.

under investigation, which is a rather narrow band compared to what one would expect a-priori. A possible explanation for this could be that, even though contemporaneous correlations can vary erratically, that these variations have a very short half-life so that conditional correlations end up being relatively smooth.

To further investigate this matter, we define the contemporaneous correlation ρ_t^c of a portfolio of assets at time t to be the value of common correlation ρ that maximizes the likelihood of observing the given standardized residuals. Formally

$$\rho_t^c = \arg\max_{\rho} \left\{ -\frac{1}{2} \left(\log |R_t| + \varepsilon_t' R(\rho)^{-1} \varepsilon_t \right) \right\}$$

where

$$R(\rho) = \begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & \ddots & & \vdots \\ \vdots & & \ddots & \rho \\ \rho & \cdots & \rho & 1 \end{pmatrix}.$$

Figure 4 shows how the conditional correlation and contemporaneous correlation evolve over time. To allow a comparison the estimate for contemporaneous correlation was smoothed using an exponentially weighted moving average with decay parameter $\alpha = 0.985$. We see that even though the contemporaneous correlation has much larger fluctuations, both correlation estimates exhibit very similar patterns over time. For example, their peaks and bottoms almost exactly coincide.

We suspect the following reason to be responsible for the different magnitudes of fluctuations in Figure 4: The average conditional correlation was obtained using a time series model that requires correlations to change slowly over time. Moreover, the likelihood function (8) for the correlation part of the model severely penalizes (via R_t^{-1}) high estimated values of correlation at a certain point in time t if the realized (or contemporaneous) correlation is low. In a nutshell, the likelihood function (8) heavily penalizes overestimating correlations and only moderately penalizes underestimating correlations, which causes the DCC correlation estimates to be *(i)* downward biased and *(ii)* to only change slowly over time. On the other hand, the estimates of contemporaneous correlation ρ_t^c were obtained separately for each point in time and can therefore *(i)* take on some rather extreme values as can be seen in Figure 5 and *(ii)* fluctuate erratically over time[7].

The observations above lead us to conclude that a dynamic conditional correlation model, given by (4) and (5), is probably not suitable for assessing the downside risk of a large portfolio of stocks, since it puts too much emphasis on fitting the correlation dynamics during quiet times as opposed to times when volatilities and correlations are

[7]For example, it is not uncommon to see contemporaneous correlation to drop form 80% to 20% and then go back up to 80% on the next day.

high. If the model was used for Value-at-Risk calculations, it would presumably underestimate e.g. the 1% and 5% VaR. We leave a more detailed treatment of this area open for future research.

5 Conclusion

This paper introduces a model for asset returns that incorporates joint heteroscedasticity as well as time varying correlations. Applying this model to ten years of data for stocks in the Dow Jones Industrial Average shows that the average conditional pairwise return correlation behaves strongly countercyclical. This means that the benefits of diversification go down exactly when they are most desirable, which might serve as an explanation for the volatility smile in index options vs. individual stocks options.

We have seen that the estimates for correlation of the DCC(1,1) model tend to be downward biased since this model puts too much emphasis on fitting the data during quite times. A different model specification for correlations might therefore be needed, for example (9) and (10), to produce accurate VaR calculations.

Finally, our findings suggest that either average conditional correlation or a smoothed version of contemporaneous correlation could be used as a business cycle or sentiment indicator. The most frequently used business cycle indicators (e.g. GDP growth, consumption, unemployment rate, car sales, . . .), rely on monthly or quarterly data and are often reported with a delay. On the other hand, the correlation estimators can be calculated on a daily basis and (similar to implied volatility for options) might contain information about agent's expectations about future states of the economy. We leave assessing the performance of such an indicator open for future research.

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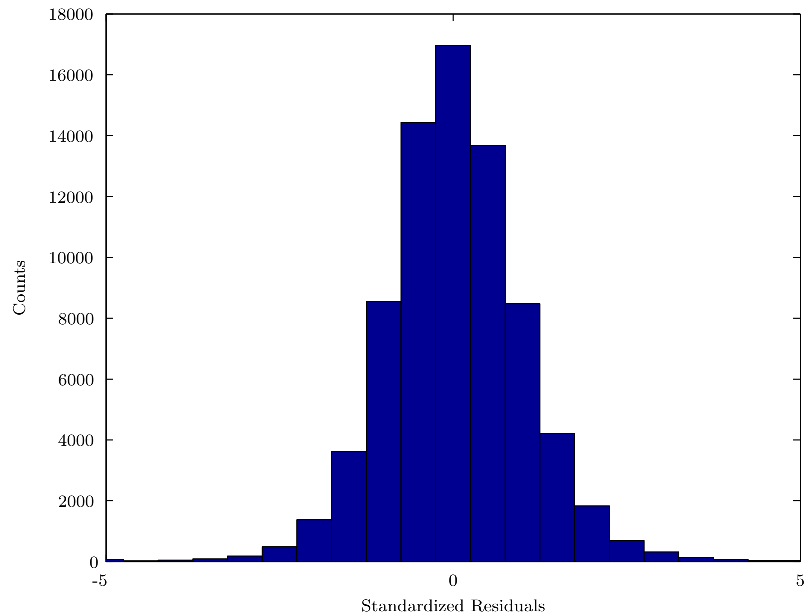

Figure 1: Histogram of the standardized residuals ε_t^i for all stocks in the Dow Jones Industrial Average

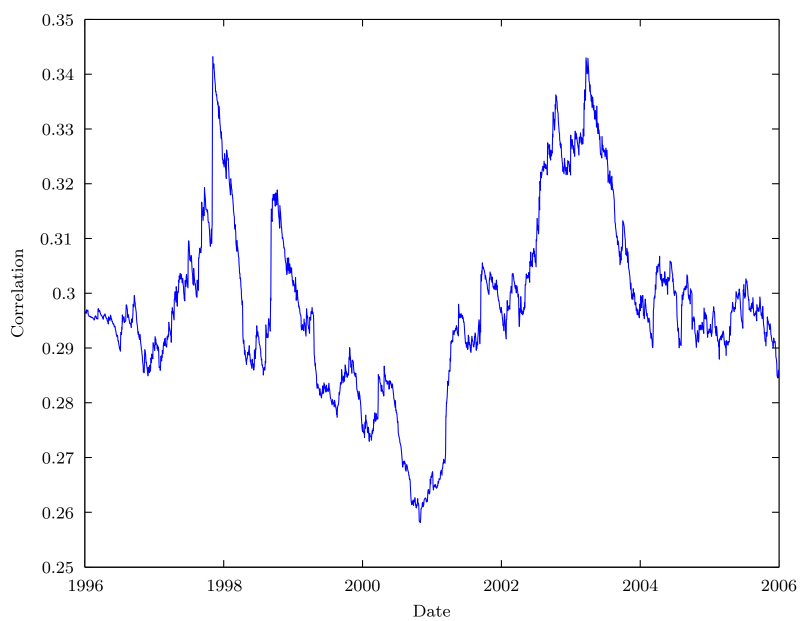

Figure 2: Average conditional pairwise correlation of all companies in the Dow Jones Industrial Average.

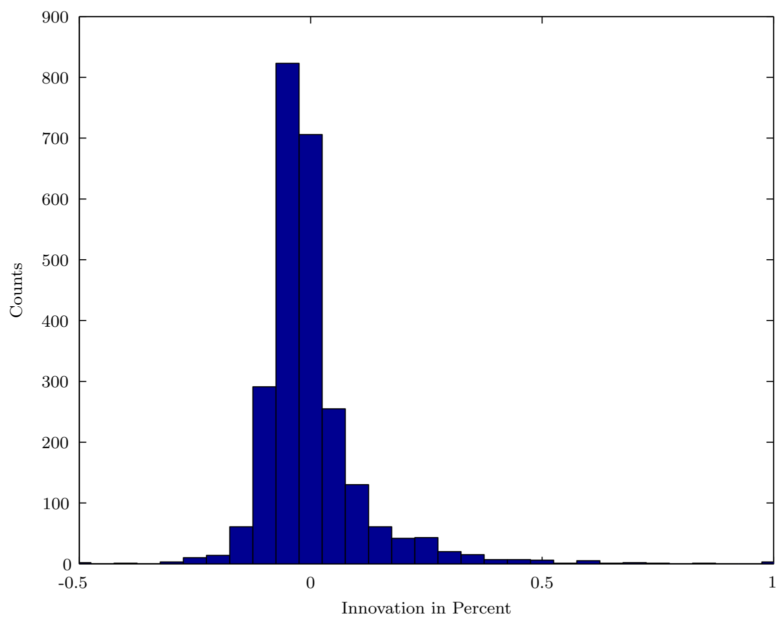

Figure 3: Histogram of innovations to the average conditional pairwise correlation $\bar{\rho}_t$ of stocks in the Dow Jones Industrial Average

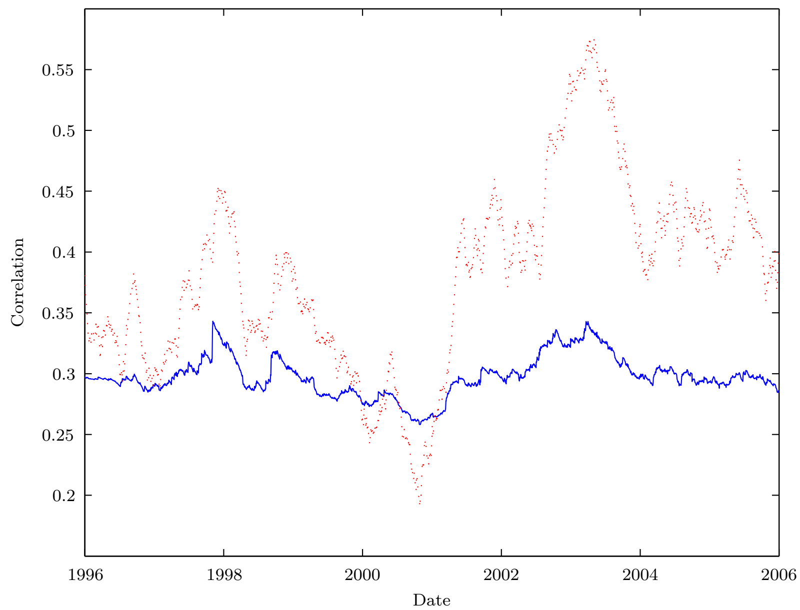

Figure 4: The solid line shows the average conditional pairwise correlation, the dotted line a smoothed version (using an exponentially weighted moving average with $\alpha = 0.98$) of contemporaneous correlation.

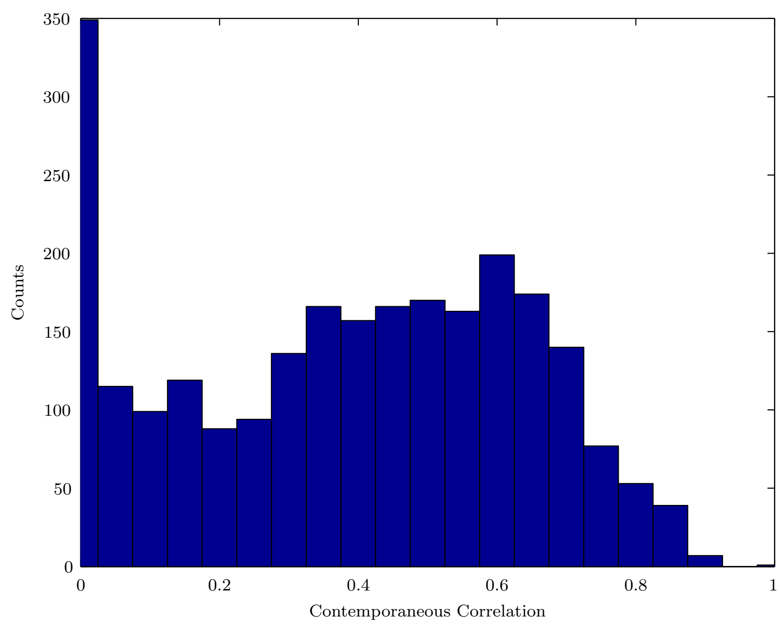

Figure 5: Histogram of the contemporaneous correlation ρ_t^c

Company	Ticker	DJIA Weight	Market Cap $bn
3M Co.	MMM	5.29%	55.8
Alcoa Inc.	AA	2.21%	26.5
Altria Group Inc.	MO	5.24%	150.2
American Express Co.	AXP	3.90%	68.1
American International Group Inc.	AIG	4.76%	173.8
AT&T Inc.	T	2.03%	91.8
Boeing Co.	BA	5.33%	58.7
Caterpillar Inc.	CAT	5.42%	50.1
Citigroup Inc.	C	3.33%	233.3
Coca-Cola Co.	KO	3.04%	100.0
E.I. DuPont de Nemours & Co.	DD	2.98%	37.7
Exxon Mobil Corp.	XOM	4.43%	370.6
General Electric Co.	GE	2.40%	350.2
General Motors Corp.	GM	1.40%	11.3
Hewlett-Packard Co.	HPQ	2.42%	90.3
Home Depot Inc.	HD	3.06%	89.6
Honeywell International Inc.	HON	3.03%	35.2
Intel Corp.	INTC	1.48%	122.8
International Business Machines Corp.	IBM	5.81%	125.8
Johnson & Johnson	JNJ	4.17%	171.8
JPMorgan Chase & Co.	JPM	3.02%	145.0
McDonald's Corp.	MCD	2.53%	44.5
Merck & Co. Inc.	MRK	2.56%	76.8
Microsoft Corp.	MSFT	1.96%	275.2
Pfizer Inc.	PFE	1.90%	194.4
Procter & Gamble Co.	PG	4.33%	202.3
United Technologies Corp.	UTX	4.20%	59.8
Verizon Communications Inc.	VZ	2.44%	98.4
Wal-Mart Stores Inc.	WMT	3.29%	189.2
Walt Disney Co.	DIS	2.05%	53.9

Table 1: Composition of the Dow Jones Industrial Average as of March 6th, 2006.

Ticker	κ_i	std(κ_i)	λ_i	std(λ_i)	c_i	std(c_i)	t-statistic for c_i
MMM	0.024	0.006	0.951	0.010	0.030	0.009	3.316
AA	0.069	0.014	0.810	0.030	0.125	0.029	4.315
MO	0.058	0.009	0.863	0.015	0.069	0.012	5.592
AXP	0.074	0.013	0.721	0.047	0.238	0.058	4.137
AIG	0.040	0.014	0.807	0.029	0.181	0.032	5.668
T	0.050	0.010	0.948	0.011	0.006	0.004	1.386
BA	0.039	0.009	0.903	0.026	0.052	0.021	2.503
CAT	0.044	0.012	0.843	0.018	0.126	0.018	7.150
C	0.048	0.011	0.902	0.027	0.040	0.017	2.365
KO	0.057	0.009	0.913	0.010	0.040	0.008	5.133
DD	0.040	0.012	0.894	0.023	0.083	0.023	3.543
XOM	0.006	0.005	0.930	0.016	0.127	0.032	4.027
GE	0.104	0.019	0.748	0.033	0.195	0.032	6.053
GM	0.096	0.015	0.831	0.025	0.086	0.020	4.356
HPQ	0.040	0.008	0.942	0.012	0.038	0.015	2.608
HD	0.063	0.009	0.912	0.012	0.029	0.009	3.082
HON	0.085	0.013	0.877	0.026	0.027	0.012	2.282
INTC	0.061	0.012	0.894	0.025	0.032	0.012	2.659
IBM	0.039	0.007	0.924	0.012	0.033	0.008	4.327
JNJ	0.090	0.024	0.657	0.052	0.185	0.031	5.945
JPM	0.051	0.009	0.848	0.032	0.114	0.031	3.661
MCD	0.056	0.017	0.313	0.040	0.807	0.057	14.193
MRK	0.076	0.012	0.867	0.032	0.069	0.029	2.405
MSFT	0.098	0.018	0.848	0.030	0.065	0.022	3.021
PFE	0.051	0.007	0.943	0.008	0.007	0.003	2.094
PG	0.052	0.009	0.933	0.011	0.015	0.006	2.457
UTX	0.054	0.016	0.755	0.037	0.164	0.030	5.570
VZ	0.092	0.014	0.859	0.021	0.045	0.012	3.659
WMT	0.068	0.011	0.882	0.019	0.050	0.014	3.527
DIS	0.060	0.010	0.909	0.017	0.020	0.008	2.616

Table 2: Fitted coefficients for the volatility part of the model.

α	std(α)	β	std(β)
0.0031	0.0003	0.9830	0.0021

Table 3: Fitted coefficients for the correlation part of the model.